

Topic 08 - Straight Lines (Solutions)

Q1, (Jun 2005, Q4)

A line has equation $3x + 5y = 12$. Find its gradient and the coordinates of the points where it crosses the axes. [4]

$$\Rightarrow 5y = 12 - 3x \Rightarrow y = \frac{12}{5} - \frac{3}{5}x \Rightarrow \text{Gradient} = -\frac{3}{5}$$

$$\text{Let } x = 0 \Rightarrow y\text{-intercept} = \frac{12}{5} \therefore (0, \frac{12}{5})$$

$$\text{Let } y = 0 \Rightarrow 3x + 5(0) = 12 \Rightarrow x = 4 \therefore (4, 0)$$

Q2, (Jun 2006, Q3)

Find the coordinates of the point of intersection of the lines $y = 3x + 1$ and $x + 3y = 6$. [3]

$$\begin{aligned} y = 3x + 1 &\Rightarrow x + 3(3x + 1) = 6 \\ &\Rightarrow x + 9x + 3 = 6 \\ &\Rightarrow 10x = 3 \Rightarrow x = \frac{3}{10} \end{aligned}$$

$$\text{Let } x = \frac{3}{10} \Rightarrow y = 3\left(\frac{3}{10}\right) + 1 = \frac{19}{10} \therefore \left(\frac{3}{10}, \frac{19}{10}\right)$$

Q3, (Jan 2007, Q1)

Find, in the form $y = ax + b$, the equation of the line through $(3, 10)$ which is parallel to $y = 2x + 7$. [3]

$$m = 2 \quad (x_1, y_1) = (3, 10)$$

$$\begin{aligned} \Rightarrow y - 10 &= 2(x - 3) \Rightarrow y - 10 = 2x - 6 \\ &\Rightarrow y = 2x + 4 \end{aligned}$$

Q4, (Jan 2008, Q5)

(i) Find the gradient of the line $4x + 5y = 24$. [2]

(ii) A line parallel to $4x + 5y = 24$ passes through the point $(0, 12)$. Find the coordinates of its point of intersection with the x -axis. [3]

$$\text{i/ } 5y = 24 - 4x \Rightarrow y = \frac{24}{5} - \frac{4}{5}x \therefore m = -\frac{4}{5}$$

$$\text{ii/ } m = -\frac{4}{5}, c = 12 \Rightarrow y = -\frac{4}{5}x + 12$$

$$\text{Let } y = 0 \Rightarrow 0 = -\frac{4}{5}x + 12 \Rightarrow \frac{4}{5}x = 12 \Rightarrow x = 15 \therefore (15, 0)$$

Q5, (Jun 2008, Q2)

(i) Find the points of intersection of the line $2x + 3y = 12$ with the axes. [2]

(ii) Find also the gradient of this line. [2]

i/ y-intercept: Let $x = 0 \Rightarrow 3y = 12 \Rightarrow y = 4 \therefore (0, 4)$

x-intercept: Let $y = 0 \Rightarrow 2x = 12 \Rightarrow x = 6 \therefore (6, 0)$

ii/ $3y = 12 - 2x \Rightarrow y = 4 - \frac{2}{3}x \therefore m = -\frac{2}{3}$

Q6, (Jun 2008, Q12i)

Find the equation of the line passing through A (-1, 1) and B (3, 9). [3]

$$m = \frac{9 - 1}{3 - (-1)} = \frac{8}{4} = 2$$

$$\therefore y - 9 = 2(x - 3) \Rightarrow y - 9 = 2x - 6 \Rightarrow y = 2x + 3$$

Q7, (Jan 2009, Q2)

Find the equation of the line passing through (-1, -9) and (3, 11). Give your answer in the form $y = mx + c$. [3]

$$m = \frac{11 - (-9)}{3 - (-1)} = \frac{20}{4} = 5$$

$$y - 11 = 5(x - 3) \Rightarrow y - 11 = 5x - 15$$

$$\Rightarrow y = 5x - 4$$

Q8, (Jun 2009, Q1)

A line has gradient -4 and passes through the point (2, 6). Find the coordinates of its points of intersection with the axes. [4]

$$m = -4 \Rightarrow y - 6 = -4(x - 2)$$

$$\Rightarrow y - 6 = -4x + 8$$

$$\Rightarrow y = -4x + 14$$

$$\therefore y\text{-int} = (0, 14)$$

$$x\text{-int}: \text{Let } y = 0 \Rightarrow -4x + 14 = 0$$

$$\Rightarrow 4x = 14 \Rightarrow x = \frac{7}{2}$$

$$\therefore \left(\frac{7}{2}, 0\right)$$

Q9, (Jan 2010, Q3)

(i) Find the coordinates of the point where the line $5x + 2y = 20$ intersects the x -axis. [1]

(ii) Find the coordinates of the point of intersection of the lines $5x + 2y = 20$ and $y = 5 - x$. [3]

i/ Let $y = 0 \Rightarrow 5x = 20 \Rightarrow x = 4 \therefore (4, 0)$

ii/ Since $y = 5 - x \Rightarrow 5x + 2(5 - x) = 20$
 $\Rightarrow 5x + 10 - 2x = 20$
 $\Rightarrow 3x = 10 \Rightarrow x = \frac{10}{3}$

$\Rightarrow y = 5 - \frac{10}{3} = \frac{5}{3} \therefore \left(\frac{10}{3}, \frac{5}{3}\right)$

Q10, (Jun 2010, Q1)

Find the equation of the line which is parallel to $y = 3x + 1$ and which passes through the point with coordinates $(4, 5)$. [3]

$m = 3 \Rightarrow y - 5 = 3(x - 4)$

$\Rightarrow y - 5 = 3x - 12$

$\Rightarrow y = 3x - 7$